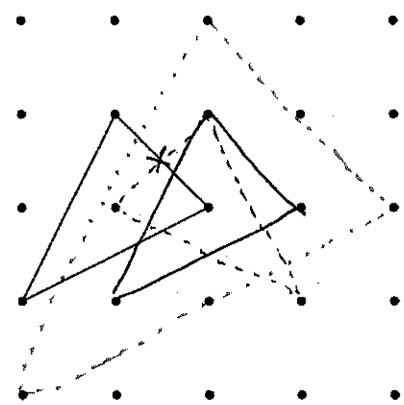
MA 202 Exam 3 Review Worksheet

Answer Key

1. A triangle is formed on the geoboard with a rubber band.



(a) How many different translations of the triangle are possible (including the original triangle)? The entire triangle must remain on the geoboard.

$\boxed{9}$ The triangle can be moved up or down by 1 peg or to the right by 2 pegs, making 9 total translations.

(b) Describe a sequence of horizontal and vertical translations that move the triangle so that the middle peg is in the center of the triangle. Draw the translated triangle on the above geoboard. Is it possible to rotate the triangle so that the middle peg is in the center of the triangle? If so, specify the center of rotation and the angle of rotation?

Translate right by 1 peg. Yes, it is possible. Rotate the triangle 90° ccw around the midpoint of its short side. (See dashed Δ above).

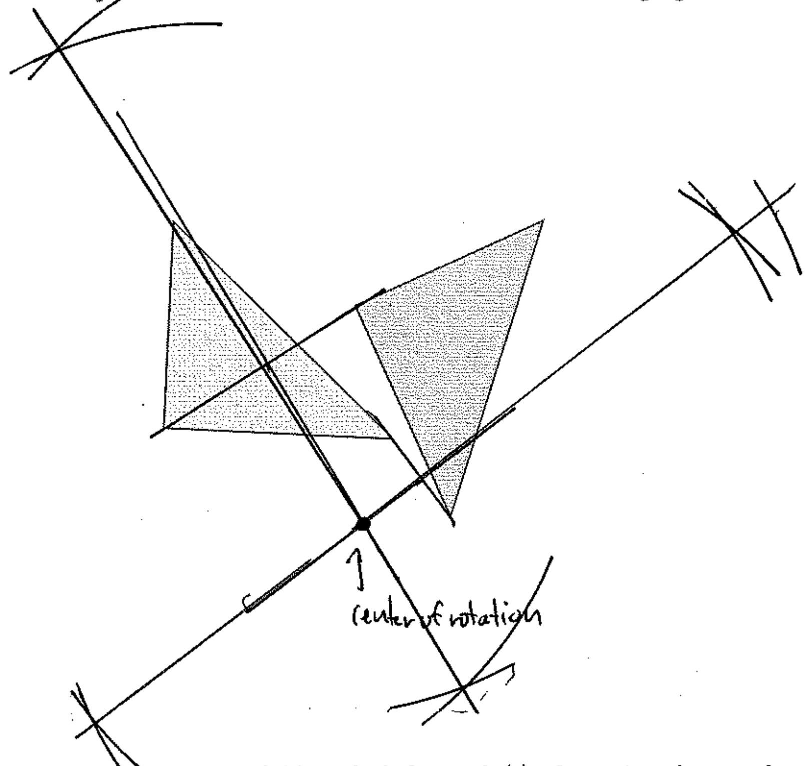
(c) Let the middle peg be the center of dilation and dilate the translated triangle from part (b) with a scale factor of 2. Draw the dilated triangle on the geoboard.

See larger dotted Δ.

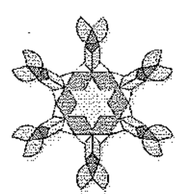
(d) How does the perimeter of the original triangle compare with the perimeter of the of the dilated triangle? How does the area of the original triangle compare with the area of the dilated triangle?

The original perimeter is $\frac{1}{2}$ of the dilated triangle's perimeter.
The original area is $\frac{1}{4}$ of the dilated triangle's area.

2. Use a straightedge and compass to find the center of rotation of the following figure:



3. Does this figure have rotational symmetry? If so, find the angle(s) of rotation that produce an image that fits exactly on the original figure.



Yes. 60°, 120°, 180°.

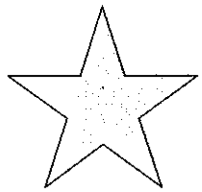
4. Identify 4 letters of the alphabet (capitalized) that are the same when reflected over a *horizontal* line. Identify 4 letters of the alphabet (capitalized) that are the same when reflected over a *vertical* line.

i) B, C, D, H

(answers could be different)

ii) A, H, X, Y

5. Describe the type of symmetry, if any, that the following objects have:



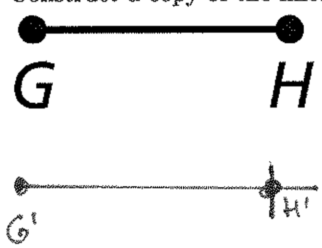
- reflective symmetry (5 lines)
- rotational symmetry (72°, 144°)

- ~~ver~~- horizontal symmetry
- vertical symmetry
- rotation by 180°

asymmetric

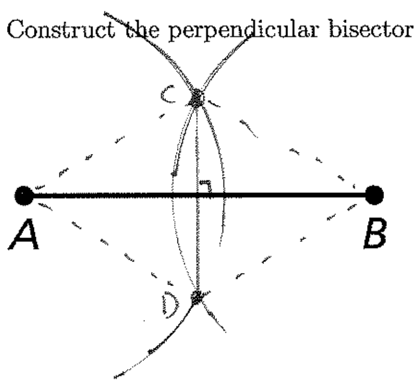
6. Use a straightedge and compass to perform each of the following constructions. Justify your technique.

(a) Construct a copy of the line.



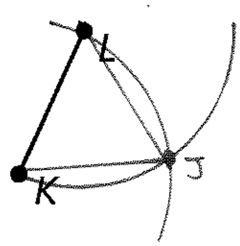
Set compass to $\overline{GH}$. Draw a ray with a point G'. ~~Keep~~. Draw an arc of length GH at G'. The intersection is H'. $\overline{G'H'} \cong \overline{GH}$ b/c compass set to GH

(b) Construct the perpendicular bisector of the line.



Set compass > $\frac{1}{2}\overline{AB}$. Draw arcs at A above and below AB. ~~The direction~~ Repeat for arcs centered at B. The line connecting the intersections of these arcs is the perpendicular bisector b/c ACBD is (by construction) a rhombus and AB, CD are its diagonals. So they are perpendicular & bisect each other.

(c) Construct an equilateral triangle whose sides have the same length as the line below.



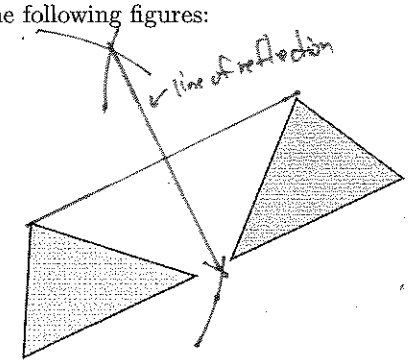
Set compass to $\overline{LK}$. Then draw arcs at K and L of radius $\overline{LK}$. Label the intersection J. $\triangle JKL$ is equilateral b/c KJ=LJ=LK by construction.

7. A student has a piece of paper in the shape of an isosceles triangle with angles of 75°, 75°, and 30°. To divide the paper into two equal parts, the student cuts down the middle of the 30° angle to the middle of the opposite side. The student says that the two pieces are congruent. Is the student correct? Explain.

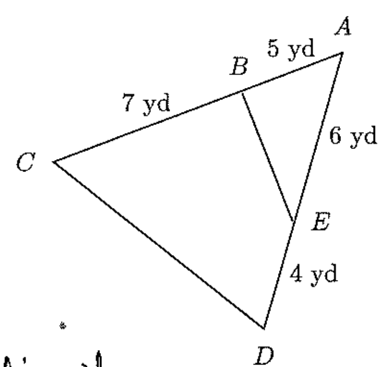
Yes. By the AAS congruence theorem, the new triangles are congruent, since each has a 15° angle, a 75° angle, and the sides between these are equal.

8. Find the line of reflection for the following figures:

Construct the perpendicular bisector of a line connecting corresponding points.



9. Determine whether the two triangles are similar. Explain your reasoning. Write the similarity statement for the triangles, if possible.



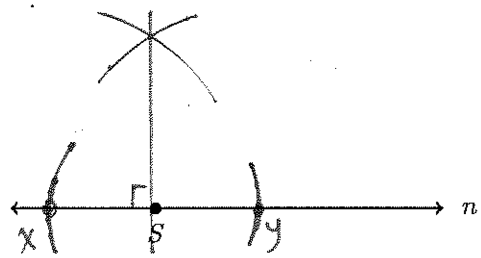
Compute ratios of corresponding sides:

longer sides: $\frac{AE}{AC} = \frac{6}{12} = \frac{1}{2}$ shorter sides: $\frac{AB}{AD} = \frac{5}{10} = \frac{1}{2}$

So by the SAS similarity theorem since $\angle A$ is shared we have $\triangle ABE \sim \triangle ADC$.

(note order of vertices)

10. Follow the procedure to construct the line perpendicular to line n through point S using only a compass and a straight edge.



(a) Draw an arc with center S that intersects n at two different points. Label these intersections as X and Y.

(b) Set the radius of the compass so that is is greater than the length of $\overline{XS}$. Draw an arc with center at X above n.

(c) Using the same radius as in Step (b), draw an arc with center at Y above n.

(d) Draw the line that passes through S and the intersections of the arcs above n.

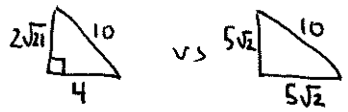
Justify this technique.

The constructed line passes through S, so we need to justify that it's perpendicular to n. But steps b and c are the construction of the perpendicular bisector of $\overline{XY}$, so the line is perpendicular to n.

11. Determine whether each of the following statements is true or false. Justify your response.

(a) All right triangles with a hypotenuse of 10 units are congruent.

False.

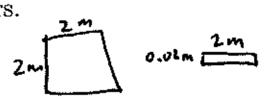


(b) All right triangles with a hypotenuse of 10 units are similar.

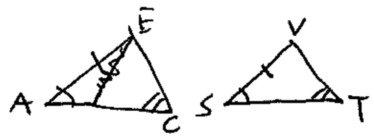
False. See example above.

(c) A square with side lengths of 2 meters is similar to a rectangle with side lengths of 2 meters and 2 centimeters.

False.



(d) If $\angle A \cong \angle S$, $\angle C \cong \angle T$, and $EA \cong VS$, then $\Delta ACE \cong \Delta STV$.



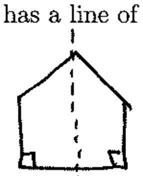
True. Use AAS congruence theorem.

(e) All isosceles triangles are similar.

False. Some isosceles triangles are acute, some are right, some are obtuse.

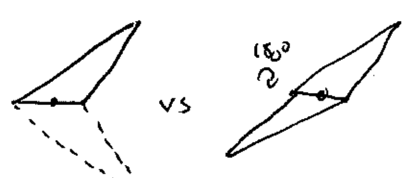
(f) If a pentagon has a line of symmetry, then it is a regular pentagon.

False.



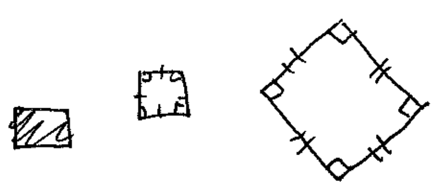
(g) Rotating a triangle by 180° around the midpoint of its base is the same transformation as reflecting it across its base.

False.



(h) Any two similar quadrilaterals can be transformed into each other with a dilation.

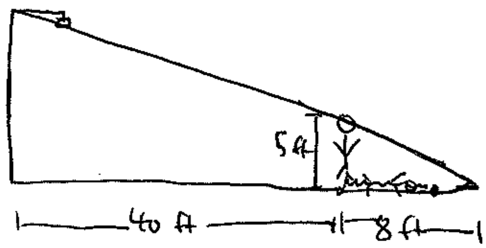
~~True~~. False.



(rotation is necessary)

12. A woman stands 40 feet from a street light and notices that her shadow cast by the street light is 8 feet long. She is 5 feet tall.

(a) Draw a diagram illustrating this scenario with all distances clearly labeled.



(b) Find two similar triangles in your diagram above and justify why they are similar.

The triangles formed by:

are similar by AA similarity since they share the angle at the tip of the shadow and both have a 90° angle.

(c) How tall is the street light?

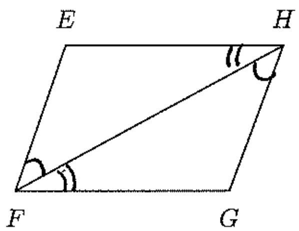
Set up a proportion from similarity:

$$\frac{h}{5} = \frac{48}{8}$$

$$\frac{h}{5} = 6$$

$$\boxed{h = 30 \text{ ft}}$$

13. When opposite angles of a quadrilateral are congruent, the quadrilateral is a parallelogram. In the figure, $\Delta EFH \cong \Delta GHF$. Determine whether quadrilateral $EFGH$ is a parallelogram.



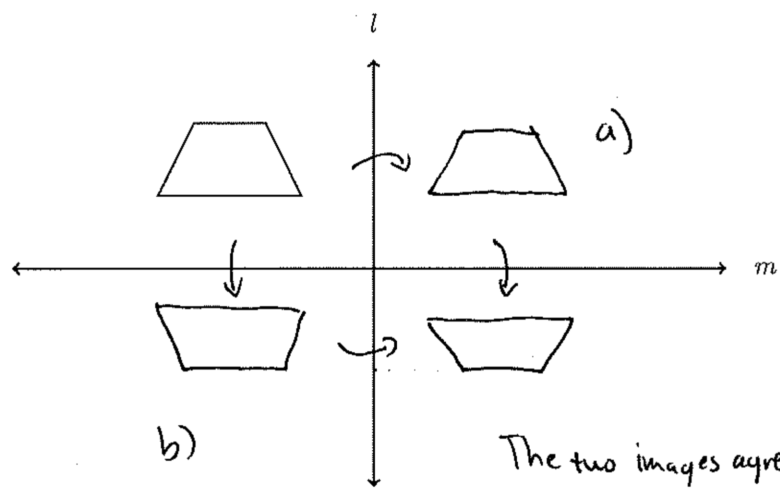
By congruence, $\angle E \cong \angle G$, and the marked pairs of angles are congruent.

Since $\angle EFG$ and $\angle EHG$ are each composed of one single marked angle and one double marked angle they must also be congruent.

So opposite angles of the quadrilateral are congruent, hence it is a parallelogram.

14. (a) Determine the location of the image of the trapezoid after it is reflected across line l and its reflection image is then reflected across line m.

(b) Determine the location of the image of the trapezoid if it is first reflected across line m and then reflected across line l.



The two images agree.